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THE EFFECT OF ERRORS IN THE QUARTER-DIFFERENCE-SQUARES MULTIPLIER

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THE EFFECT OF ERRORS

IN THE

QUARTER-DIFFERENCE-SQUARES

MULTIPLIER

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Kent W. Lawson

THE EFFECT OF ERRORS IN THE QUARTER-DIFFERENCE-SQUARES MULTIPLIER

by

Kent Winfred Lawson

Lieutenant, United States Navy

Submitted in partial fulfillment
of the requirements
for the degree of
MASTER OF SCIENCE
IN
ENGINEERING ELECTRONICS

United States Naval Postgraduate School
Monterey, California

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PREFACE

The principle of the Quarter-Difference-Squares Multiplier has long been known and was one method of obtaining the product of two quantities in the late 1800's. Tables were formed to be used with this method of multiplying, such as the one found in Chambers' seven-figure Mathematical Tables. However, the application of the Quarter-Difference-Squares Multiplier to electronic circuits has been slow due to the lack of an accurate square-law device and the tolerance requirements of the individual units that make up the multiplier.

The effect on the product output due to errors in the individual units will be investigated in this paper. The possible use of junction diodes in the square-law device will also be looked into on a theoretical basis.

The writer wishes to thank Mr. Alan Lubell of the Hughes Tool Company--Aircraft Division for his assistance and guidance in completing the present study.

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CHAPTER I

INTRODUCTION

The principle of operation of the "Quarter-Difference-Squares" Multiplier is to obtain the product by taking one-fourth the difference of the square of the sum and the square of the difference of the two quantities to be multiplied. In algebraic notion this is

$$ab = \frac{1}{4}\left[(a + b)^2 - (a - b)^2\right] \tag{1}$$

An adder and a squarer are the two basic circuits needed to perform multiplication electronically by this method. The factor of one-fourth may be considered as a gain constant. A block diagram of the "Quarter-Difference-Squares" Multiplier, hereafter referred to as the "Q-D-S" Multiplier, is shown in Figure 1.

Only sinusoidal inputs will be considered in the analysis of the output of the "Q-D-S" Multiplier because of errors in the individual units. The procedure will be to inject the error in one unit at a time and assume that all others are perfect devices. The output is then

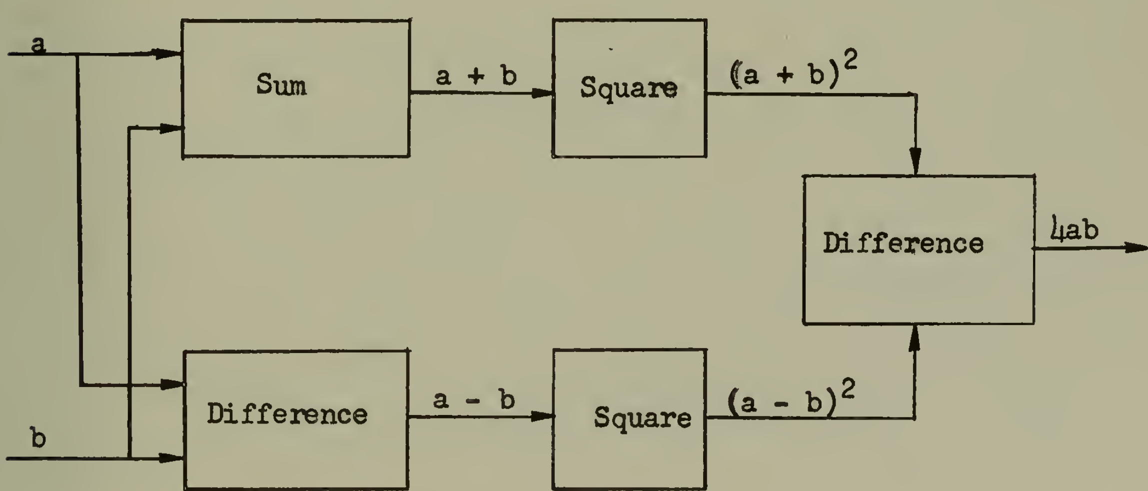

Figure 1. Block Diagram of "Q-D-S" Multiplier

compared to that of an ideal "Q-D-S" Multiplier, and the contribution to the output due to error is obtained. When injecting an error into the squaring units, a sample characteristic for a full-wave square-law rectifier using crystal diodes will be taken as the basis of error.

Because of the possible use of the "Q-S-D" Multiplier in conjunction with a 90-degree phase-difference network as a phase-sensitive detector, the use of other than square-law rectifiers will be investigated in regard to their effect on the d-c component of the output. Here again, only sinusoidal inputs will be considered. No error will be introduced into the other units.

Finally, a theoretical investigation will be made into the use of junction-type crystal diodes as the square-law devices because these diodes closely follow the basic junction equation $i_d = i_o(e^{kv_d} - 1)$, at small current levels. Here, i_d is the current through the diode, v_d is the voltage across the diode, e is the Naperian base, and i_o and k are constants dependent upon temperature, geometry, and substance of the crystal.

CHAPTER II

EFFECT OF ERRORS ON OUTPUT

1. The Ideal "Q-D-S" Multiplier.

The ideal system will be considered in conjunction with a 90-degree phase-difference network so that it will be a phase-sensitive detector with a null as the zero phase indication. Thus the inputs into the "Q-D-S" Multiplier will be

$$a = -A \cos(\omega t) \tag{2}$$

$$b = B \sin(\omega t - \varphi) \tag{3}$$

A block diagram of the phase-sensitive system is shown in Figure 2. The square-law units will be considered as ideal full-wave square-law rectifiers with the average voltages presented to the final difference unit.

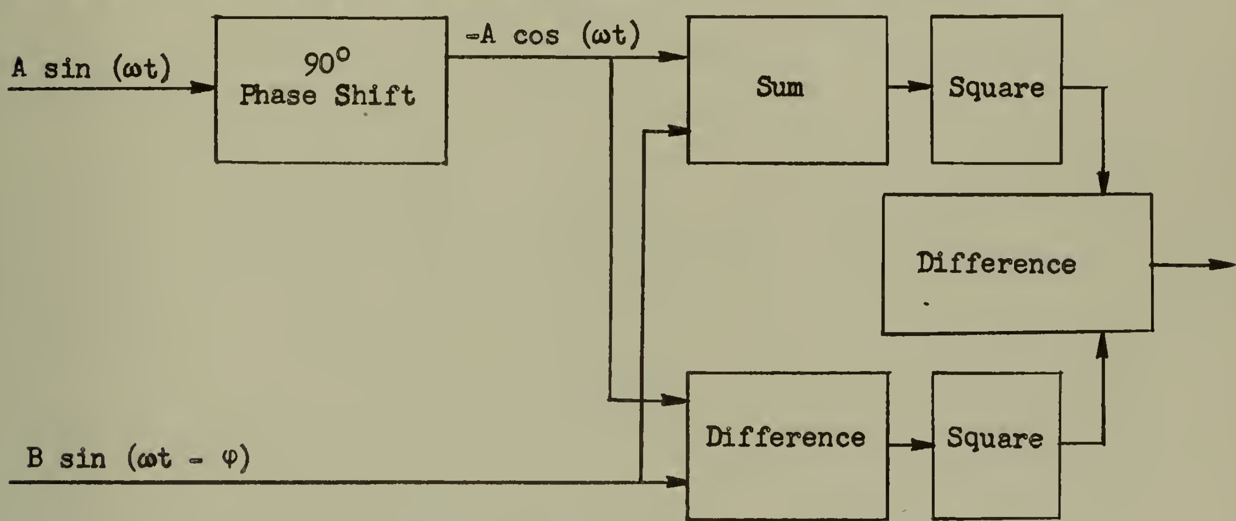

Figure 2. Phase-Sensitive System

With inputs a and b as sinusoidal voltages differing in phase and amplitude, the following relationships exist for the ideal system:

$$a = -A \cos(\omega t)$$

$$b = B \sin(\omega t - \varphi)$$

$$a + b = \sqrt{A^2 + B^2 + 2AB \sin \varphi} \quad \sin (\omega t - \beta_1) \tag{4}$$

$$\text{where } \beta_1 = \arctan \frac{A + B \sin \varphi}{B \cos \varphi}$$

$$a - b = \sqrt{A^2 + B^2 - 2AB \sin \varphi} \quad \sin (\omega t + \beta_2) \tag{5}$$

$$\text{where } \beta_2 = \arctan \frac{A - B \sin \varphi}{B \cos \varphi}$$

$$\overline{(a + b)^2} = \tfrac{1}{2}(A^2 + B^2 + 2AB \sin \varphi) \tag{6}$$

$$\overline{(a - b)^2} = \tfrac{1}{2}(A^2 + B^2 - 2AB \sin \varphi) \tag{7}$$

$$\overline{(a + b)^2} - \overline{(a - b)^2} = 2AB \sin \varphi \tag{8}$$

If the outputs of the square-law rectifiers are not averaged, the output of the final difference unit then becomes

$$(a + b)^2 - (a - b)^2 = 2AB \sin \varphi - 2AB \sin (2\omega t - \varphi) \tag{9}$$

A plot of the d-c output vs phase angle, φ, is shown in Figure 3.

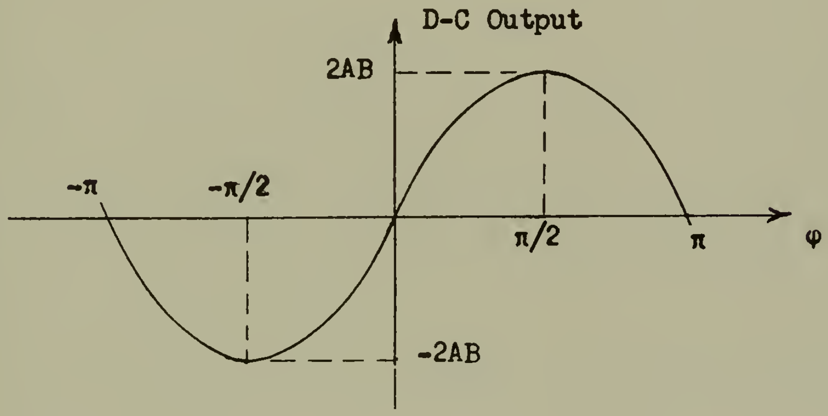

Figure 3. D-C Output of Phase-Sensitive System

2. Errors in First Sum and Difference Units.

In order to investigate the effect of errors in the first Sum and Difference units, input a will be used as the reference and assumed to be of equal magnitude and phase into each of the two units. An error of e_1

will be applied to input b in the Sum unit and an error of e_2 in the Difference unit. Of course e_1 and e_2 can be either negative or positive. Hence, the inputs to the two square-law units become, respectively,

$$a + (1 + e_1)b = (1 + e_1)B \sin(\omega t - \varphi) - A \cos(\omega t) \qquad (10)$$

and

$$a - (1 + e_2)b = -(1 + e_2)B \sin(\omega t - \varphi) + A \cos(\omega t) \qquad (11)$$

The final output, when averaging before the final Difference unit, becomes

$$2AB \sin \varphi + (e_1 + e_2)AB \sin \varphi + B^2(e_1 - e_2) + \frac{B^2}{2}(e_1^2 - e_2^2) \qquad (12)$$

It is seen from the above that the contribution to the d-c output due to errors in the first Sum and Difference units is

$$(e_1 + e_2)AB \sin \varphi + B^2(e_1 - e_2) + \frac{B^2}{2}(e_1^2 - e_2^2) \qquad (13)$$

The first term of the error contribution does not affect the null indication of zero phase but merely adds a factor to the magnitude when away from zero phase. However, the last two terms make a contribution which shifts the output curve, as shown in Figure 3, up or down depending upon the magnitude and sign of e_1 and e_2. This shift introduces a bias error which will then give a false null.

It is noted that if e_1 and e_2 are of the same magnitude and sign, no bias error is introduced. This error is trivial in that it merely indicates a change in magnitude of the input b signal.

The important effect noted from the error contribution is that the error is much less if the individual errors of the Sum and Difference units, e_1 and e_2, are of the same sign rather than of opposite sign. For example, if $e_1 = +5\%$ and $e_2 = +10\%$, the output becomes

$$2.15\ AB \sin \varphi - .0538B^2$$

which, when letting A = B = 1, corresponds to a shift in null by 1.43 degrees. If $e_1 = +5\%$ and $e_2 = -10\%$, the output becomes

$$1.95\ AB \sin \varphi + .1463B^2$$

which, when again letting A = B = 1, corresponds to a shift in null by 4.30 degrees.

3. Error in Final Difference Unit.

An error of e will be applied to the output of the difference squaring unit using the output of the sum squaring unit as the reference. Thus the final output of the "Q-D-S" Multiplier becomes

$$\overline{(a+b)}^2 - (1+e)\overline{(a-b)}^2 = 2AB \sin \varphi + eAB \sin \varphi - \frac{A^2 + B^2}{2} e$$

This equation shows that for any error in the final Difference unit, an error in magnitude -- the second term -- and a bias error -- the third term -- will exist. Here again, the amount of bias error is related to the sign of e. Using an example as before, letting A = B = 1 and e = +10%, the null is shifted by 2.73 degrees while, if e = -10%, the null is shifted by 3.02 degrees.

4. Error in the Squaring Units.

A particular response curve with a maximum output deviation of one db will be used to investigate the effect of error in the squaring units on the product output. This response curve, shown in Figure 4, is one that is typical for square-law rectifiers using non-linear devices such as germanium diodes [1] or copper oxide rectifiers [2]. It was also assumed that inputs a and b were of equal magnitude, 0.5 volt, so that the maximum combination, sum or difference, of the two inputs would be equal to

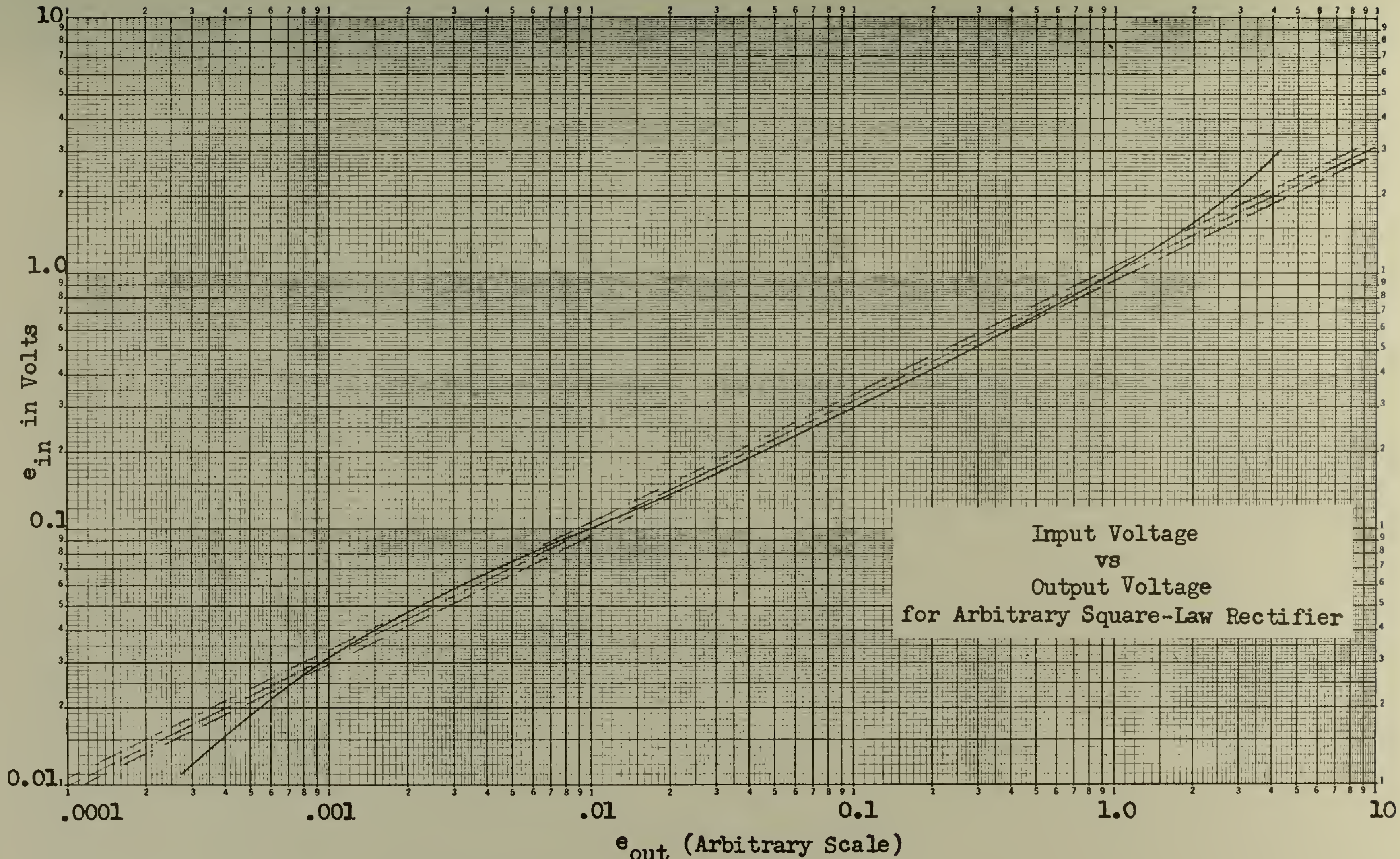

Figure 4. Response Curve of Square-Law Rectifier

one volt. One volt input into the squaring unit corresponds to the highest crossover point on the perfect square-law curve.

The point-by-point method of graphical analysis was used, and a value of output obtained for every 10 degrees of the input sine wave to each squaring unit. The value of phase angle was varied from zero to 90 degrees in ten-degree increments. Because of symmetry, the output for negative values of phase angle is the negative of that obtained for positive phase angles. The output of each squaring unit was then averaged in the same manner as would occur in a full-wave rectifier and averaging unit, and the difference of the two averages became the d-c output. Figure 5 is a plot of the d-c output of the phase-sensitive system with error in the squaring units versus phase angle. It is seen that no bias error exists and that the shape is very close to that of the ideal system. In fact the error in the output remains within 5% and is maximum at a phase angle of about 20 degrees.

5. Using Other than Square-Law Rectifiers in the "Q-D-S" Multiplier.

The possibility of using rectifiers that follow other than square-law in the phase-sensitive system will now be examined. Even though ideal cases will be assumed again, an estimate of results from other than ideal cases can be predicted from the following curves. It was seen before, when investigating the output while using an actual response curve for the squaring device, that the output contained no bias error even though the response curve varied from linear to greater than square-law and back toward linear characteristics again.

The cases of linear rectification, third-power law, fourth-power law, and fifth-power law rectification will be calculated and compared

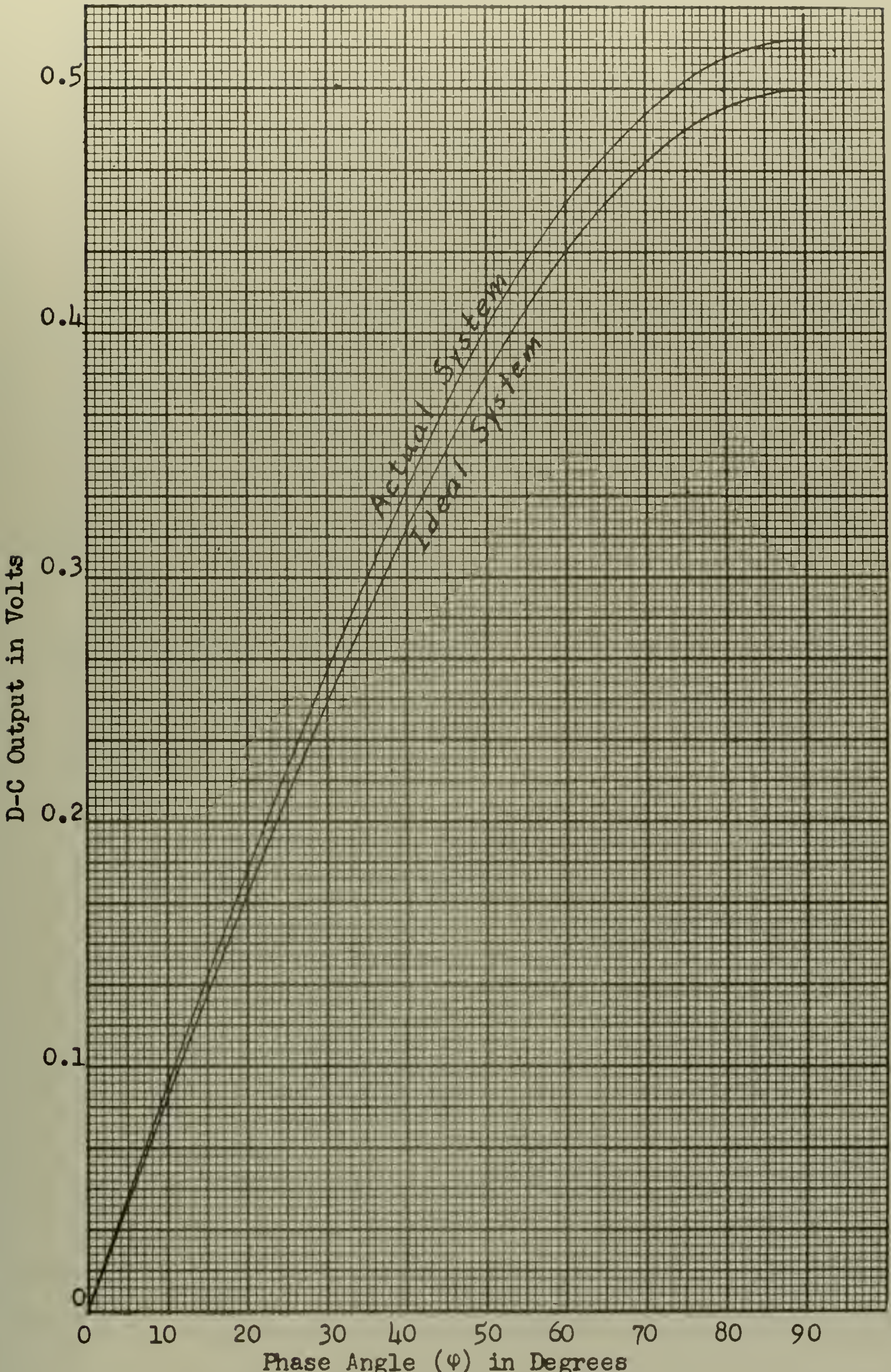

Figure 5. D-C Output of Phase-Sensitive System with Error in Squaring Units

to the output of the ideal system using square-law rectification. It will be assumed that inputs a and b are of equal magnitude and normalized to unity in order to be able to compare these cases. Averaging will be accomplished prior to the final Difference unit so that the product output will be dc, dependent only upon the input phase difference.

Carrying through the computations for each case, it is seen that the outputs are as follows:

For linear rectifiers,

$$dc = 0.9 \left[\sqrt{1 + \sin\varphi} - \sqrt{1 - \sin\varphi}\right] \tag{14}$$

For square-law rectifiers,

$$dc = 2 \sin\varphi \tag{15}$$

For third-power law rectifiers,

$$dc = 1.21 \left[(1 + \sin\varphi)^{3/2} - (1 - \sin\varphi)^{3/2}\right] \tag{16}$$

For fourth-power law rectifiers,

$$dc = 6 \sin\varphi \tag{17}$$

For fifth-power law rectifiers,

$$dc = 1.93 \left[(1 + \sin\varphi)^{5/2} - (1 - \sin\varphi)^{5/2}\right] \tag{18}$$

A plot of the various outputs versus phase angle is shown in Figure 6. Figure 7 is a similar plot with all of the curves normalized to two in order to compare their shapes to that of the ideal system. It is seen from these curves that no bias error results from the rectifiers being other than square-law and that the shape of the curves, with the exception of the linear rectifier, is very similar to that of the square-law rectifier. The output of the linear rectifier is nearly a linear variation with respect to phase angle up to 90 degrees. All of the

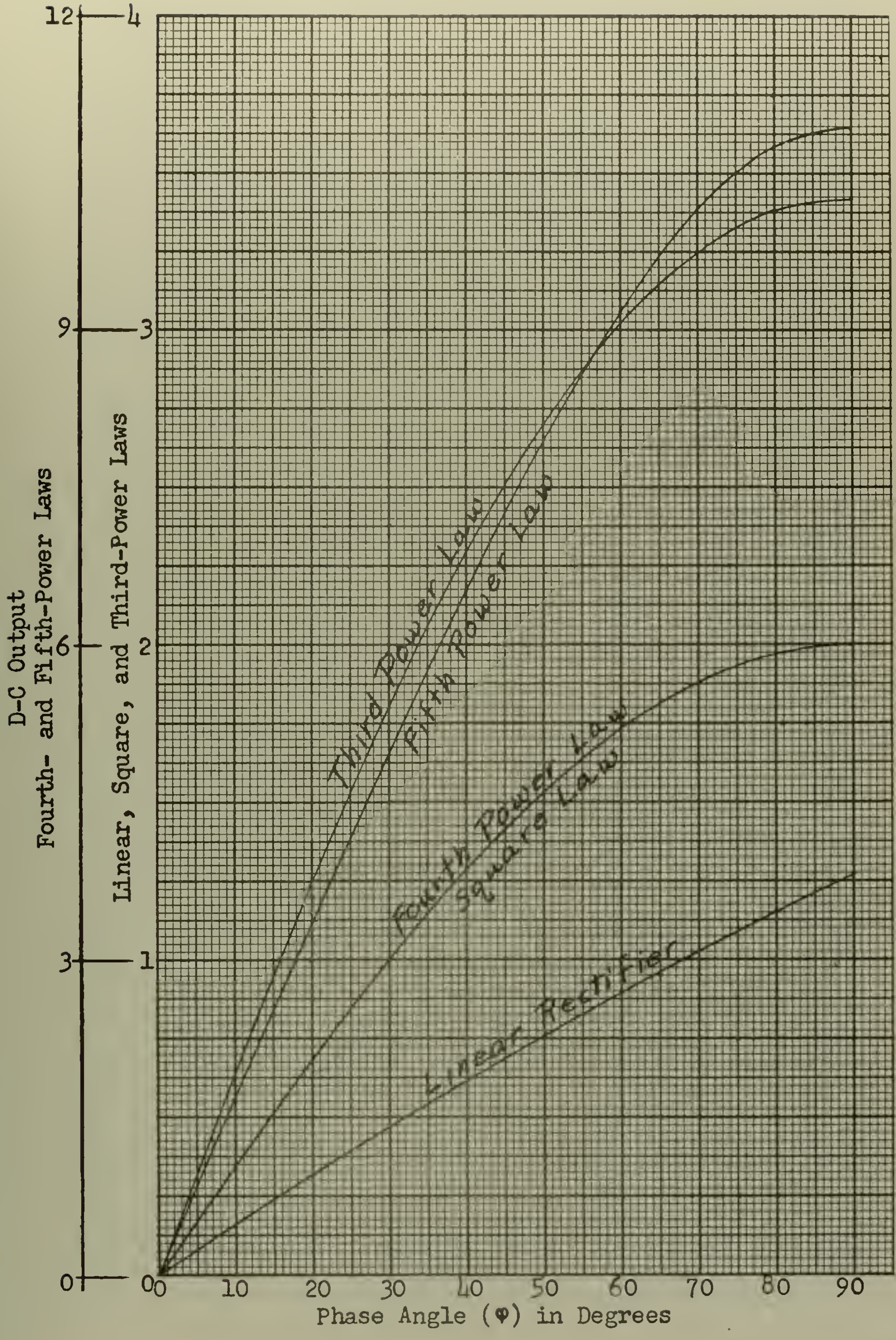

Figure 6. D-C Output of Phase-Sensitive System with Various Power-Law Rectifiers

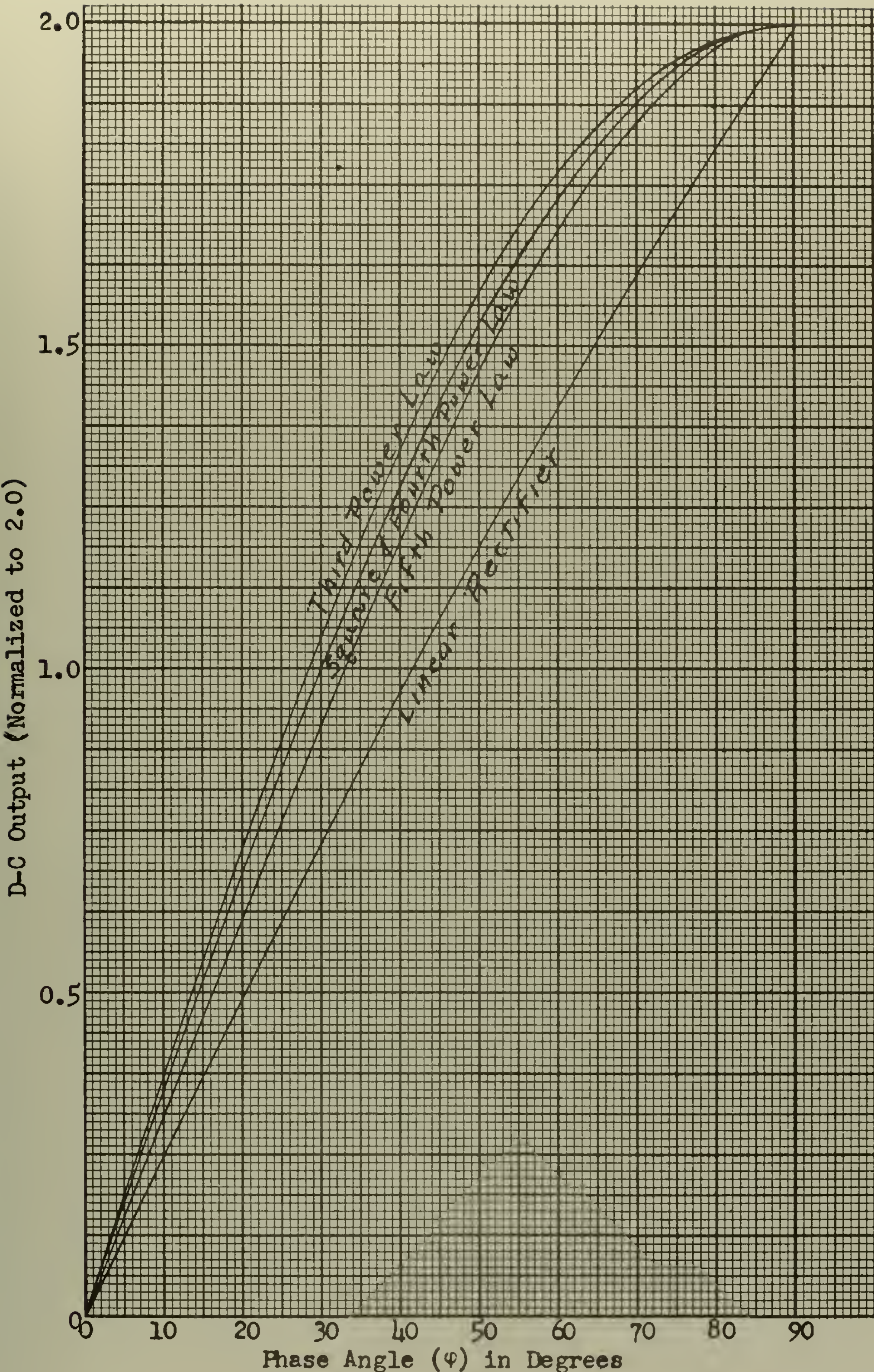

Figure 7. Normalized Curves of Figure 6

curves have even symmetry with respect to plus and minus 90 degrees, and odd symmetry with respect to zero.

It thus appears that the law of rectification is non-critical for a null-type phase indicating system. Its main effect is the magnitude of output and shape of output versus phase characteristic.

CHAPTER III

DIODES AS THE SQUARE-LAW DEVICE

At low current levels where the i_oR drop is negligible, the junction equation for semiconductors, $i_d = i_o\left[e^{k(v_d - i_dR)} - 1\right]$, can be very closely approximated by the equation, $i_d = i_o(e^{kv_d} - 1)$. Here, i_d is the forward current through the diode, v_d is the voltage across the diode, R is the ohmic resistance of the semiconductor, and i_o and k are constants.

The curve of the approximate equation is linear at minute values of current and progresses through increasing power-law as the impressed voltage, v_d, is increased. A plot of the family of curves of i_d/i_o vs v_d for various values of k is shown in Figure 8. The portion of the curve at the higher values of i_d/i_o can be made to decrease in power-law and again approach linear response by the insertion of a series resistance. A typical family of curves for k = 40 of i_d/i_o vs v_d is shown in Figure 9 for various values of R_L. The log-log plot is used because square-law, the desired response, will then be a straight line with a slope of one-half.

It is seen from Figure 9 that the i_d/i_o vs v_d response curve can be made to "S" about a true square-law curve, the deviation being proportional to range. If high accuracy of square-law response is desired, the curve can be held for only a very short range.

A circuit such as shown in Figure 10 can be used in order to extend the range of high-accuracy square-law response. This circuit has been conceived from purely theoretical considerations based on the approximate equation, $i_d = i_o(e^{kv_d} - 1)$. However, this is a very close approximation

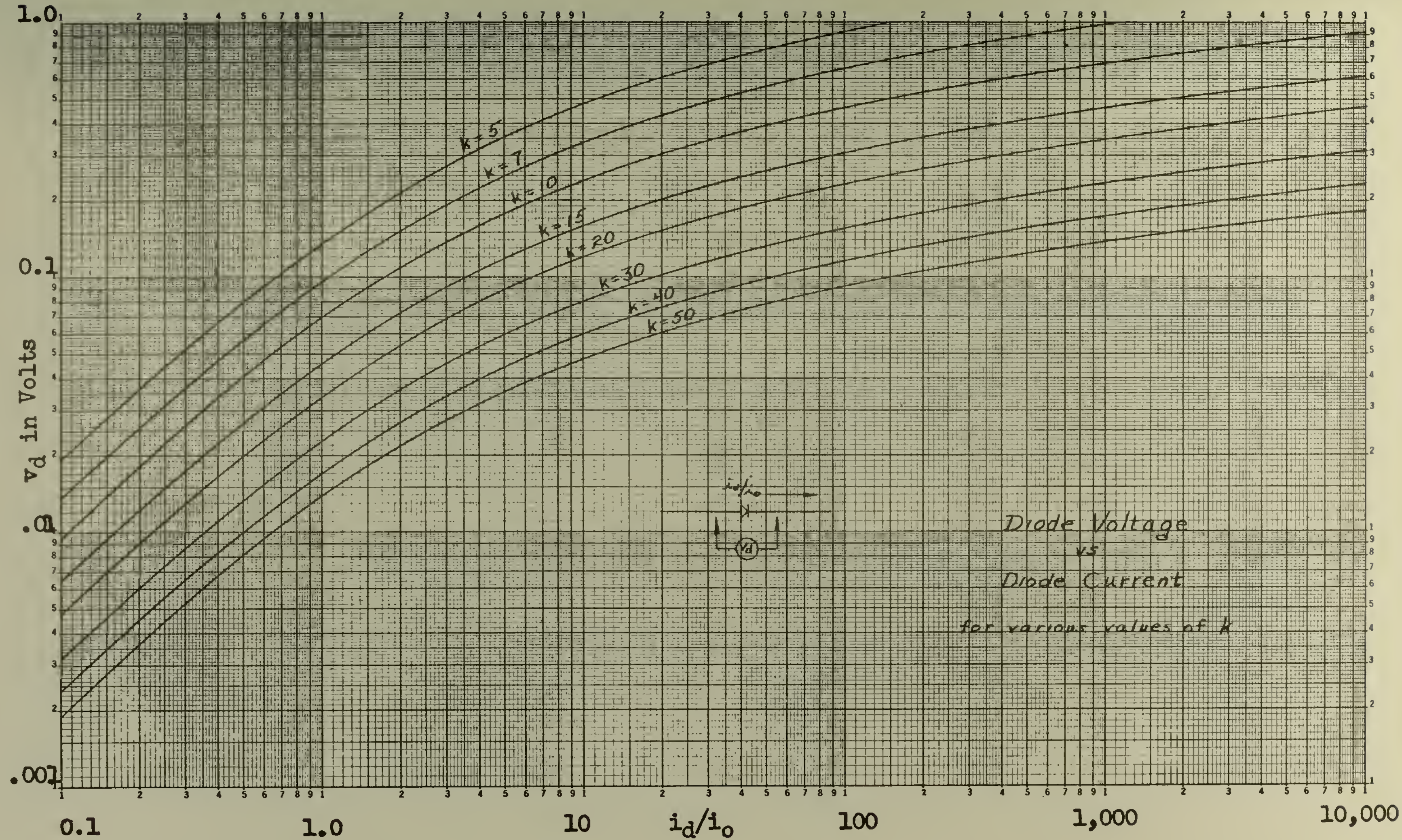

Figure 8. Family of Curves for Junction Equation for Various Values of k

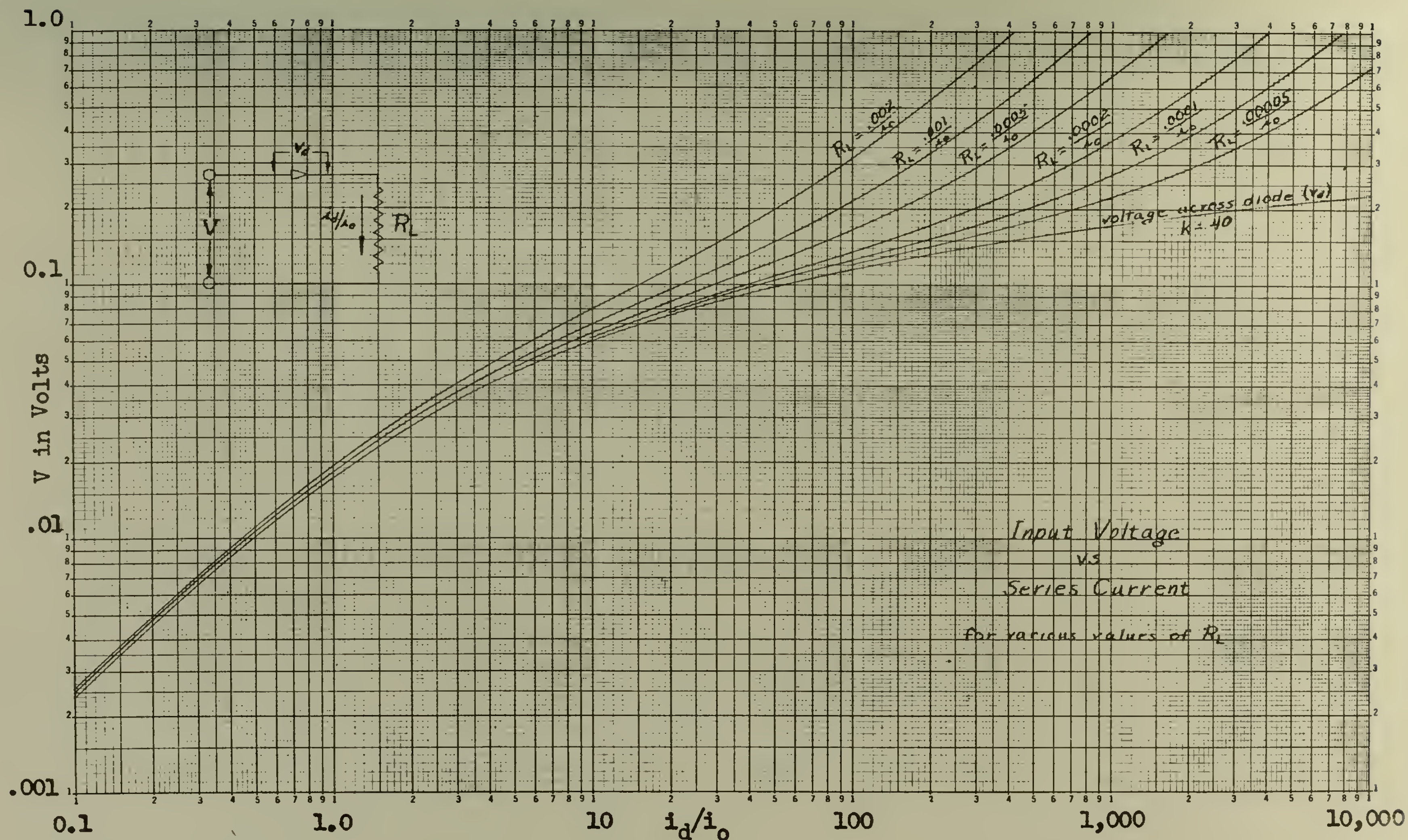

Figure 9. Family of Curves for Single Compensated Diode

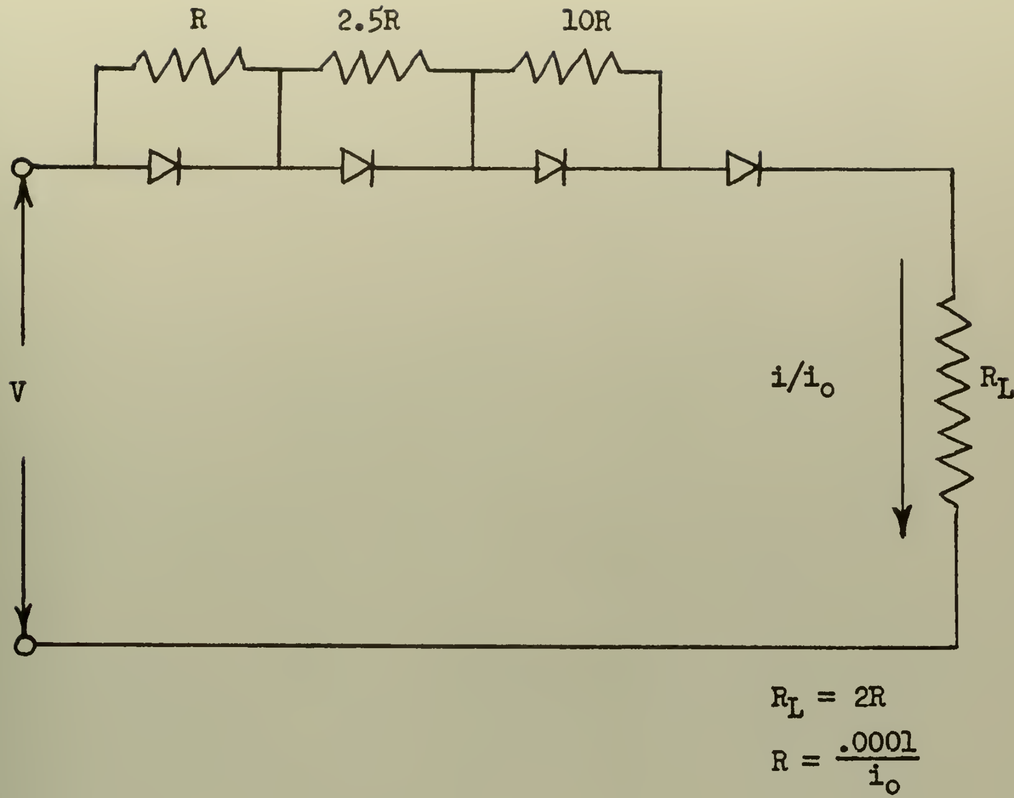

Figure 10. High-Accuracy Square-Law Circuit

for junction-type diodes because of the very low value of ohmic resistance of the semiconductor.

Figure 11 shows the response of the square-law device for diodes whose k = 40. It should be noted also that the values of resistance and current are in terms of i_o, a constant of the diode. A value of k = 40 is used because it is approximately the theoretical value at room temperature. Actually, k may vary from 40 to 5 depending on the diode. Thus, there are two constants to be considered in order to determine an actual circuit, but the response curve will maintain the shape of the curve in Figure 11. Varying the value of k will move the curve in a vertical direction, with decreasing values of k displacing it upward.

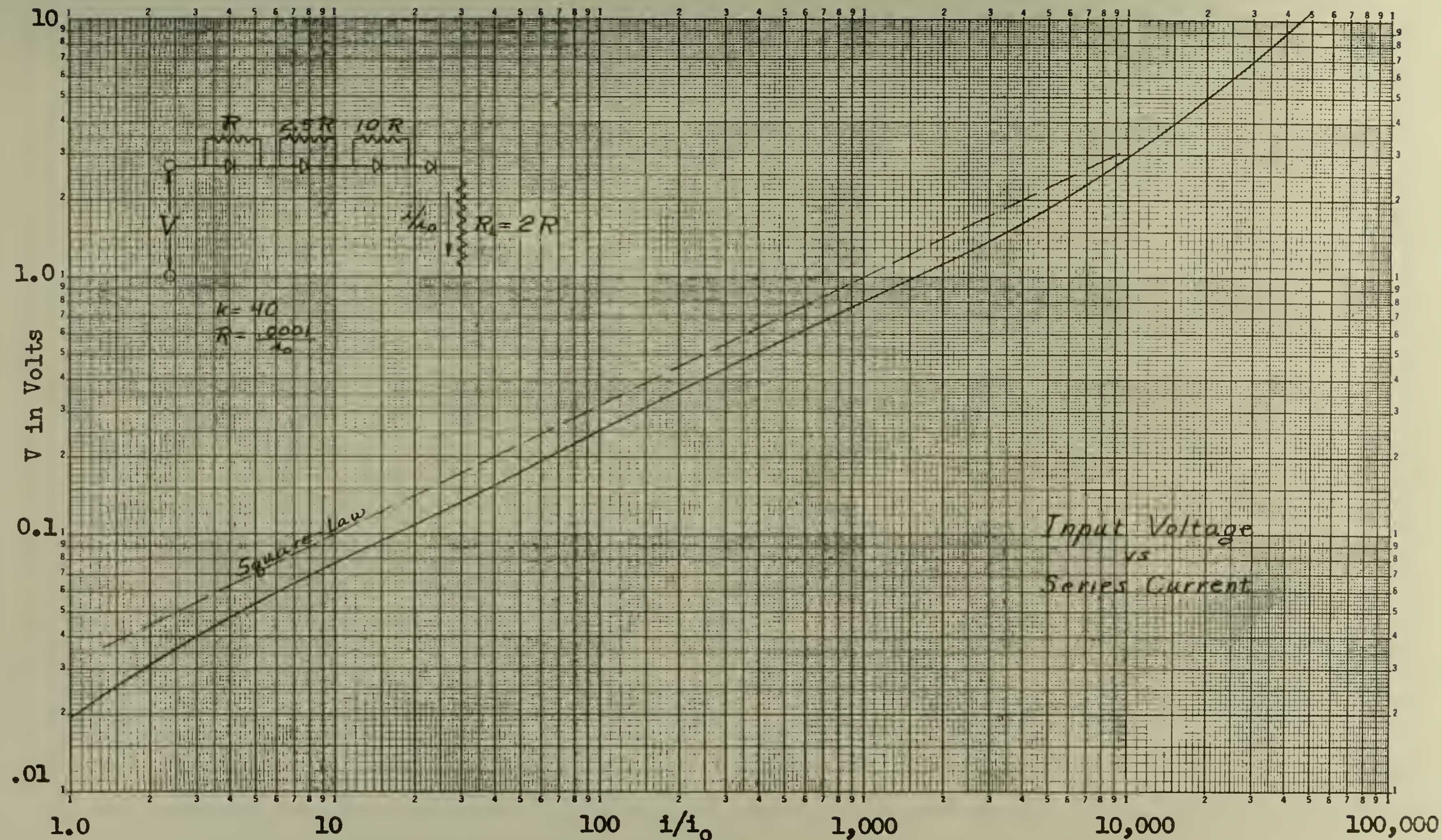

Figure 11. Response Curve of Square-Law Circuit

The value of i_o will determine the actual values of the shunt and load resistances and the current.

The effect of the diodes on the response characteristic is staggered by shunting the diodes with differing values of resistance. The square-law range is thus extended over that of a single diode with a compensating resistance. Figure 12 shows the contribution in percentage of each of the series elements to the total voltage around the circuit. By using the values as shown in Figure 10, the square-law response is very accurate over a range of about 30 to 1 of input voltage, or 900 to 1 of output.

Accurate data on the diodes must first be obtained in order to use this method of obtaining square-law response. The four diodes must be of identical characteristics, that is, have the same values of k and i_o. The value of k and i_o can be obtained by measuring the voltage across the diode and its current and plotting the values of log-log paper of the same scale as that of Figure 8. The value of k is determined by sliding the curve along identical voltage lines until the measured curve coincides with one of the family of k-curves (interplation may be used if needed). Also when matched, the value of i_o is determined from the measured value of current and the coinciding value of i_d/i_o. When the value of i_o is known, the values of the three shunt resistors can be calculated and the total series resistance determined.

The table shown below contains computed values of the circuit parameters that will help locate the diode characteristics and load resistances desired. The following three examples have been worked out by interpolation of the specific data from this table.

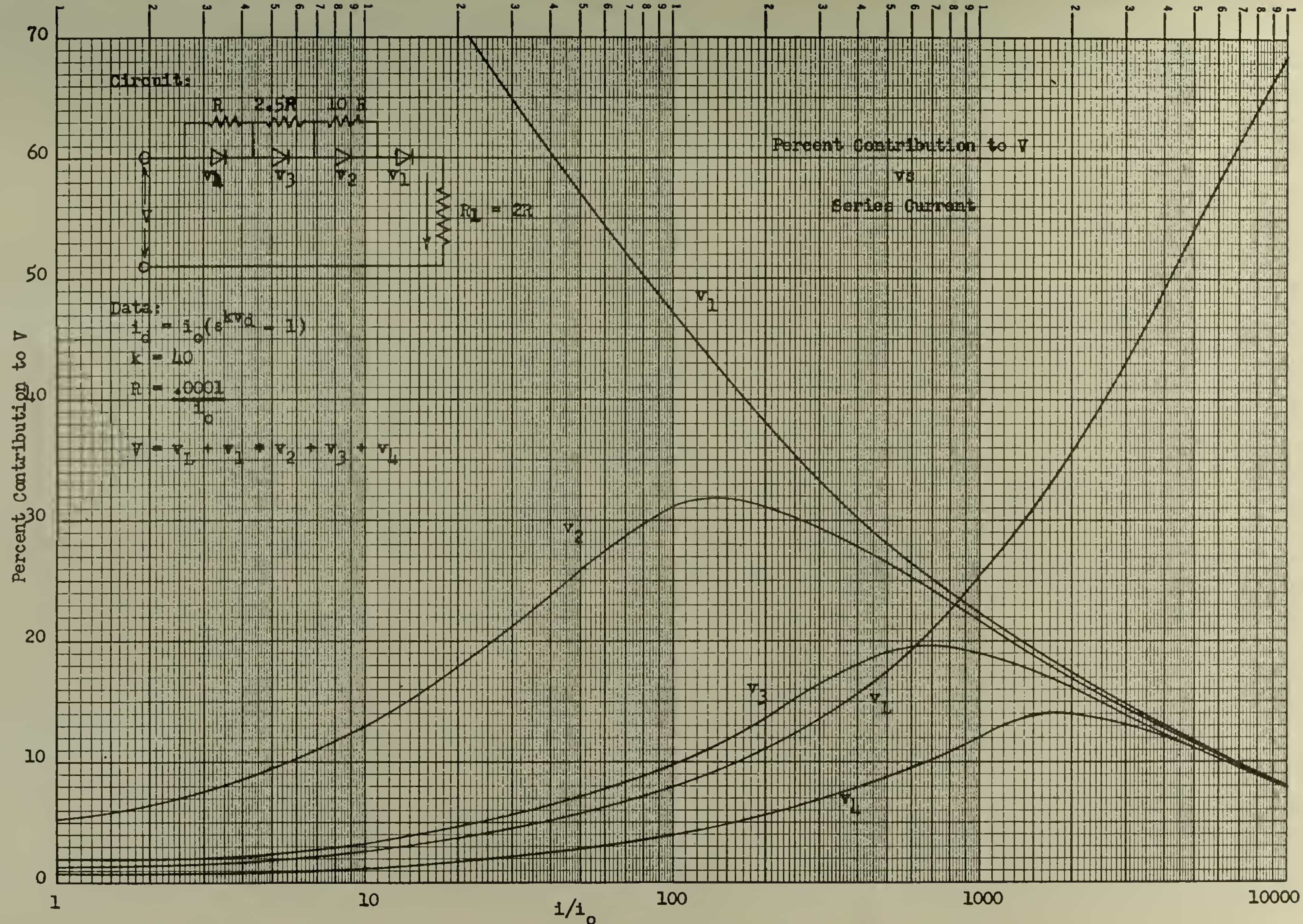

Figure 12. Contributions of Series Elements in Square-Law Circuit

i_o (μa)	i_{max} (μa)	$\bar{i}_{max}$ (μa)	R k=40	k=30	k=20	k=15	k=10	k=7	k=5
0.0001	0.3	0.15	1M	1.33M	2M	2.67M	4M	5.17M	8M
0.0002	0.6	0.30	500K	667K	1M	1.33M	2M	2.87M	4M
0.0005	1.5	0.75	200K	267K	400K	533K	800K	1.14M	1.6M
0.001	3.0	1.5	100K	133K	200K	267K	400K	571K	800K
0.002	6.0	3.0	50K	66.7K	100K	133K	200K	287K	400K
0.005	15	7.5	20K	26.7K	40K	53.3K	80K	114K	160K
0.01	30	15	10K	13.3K	20K	26.7K	40K	57.1K	80K
0.02	60	30	5K	6.67K	10K	13.3K	20K	28.7K	40K
0.05	150	75	2K	2.67K	4K	5.33K	8K	11.4K	16K
0.1	300	150	1K	1.33K	2K	2.67K	4K	5.71K	8K

k	V_{max} (Volts)
40	1.38
30	1.84
20	2.76
15	3.68
10	5.52
7	7.89
5	11.04

Tables of Values for Circuit in Figure 10

(i_{max} is the maximum instantaneous direct current;

$\bar{i}_{max}$ is the maximum average direct current)

Three examples of using the Tables on page 21 in determining circuit parameters:

Example No. 1

Given: $\bar{i}_{max} = 100\ \mu a$ From Table: $i_o = .067\ \mu a$

$V_{max} = 3$ volts $k = 18.4\ \text{volts}^{-1}$

$R = 3.26K$

$R_L = 6.52K$

Example No. 2

Given: $\bar{i}_{max} = 50\ \mu a$ From Table: $i_o = .033\ \mu a$

$R_L = 12K$ $k = 20\ \text{volts}^{-1}$

$R = 6K$

$V_{max} = 2.76$ volts

Example No. 3

Given: $R_L = 15K$ From Table: $k = 27.6\ \text{volts}^{-1}$

$V_{max} = 2$ volts $R = 7.5K$

$i_o = .0193\ \mu a$

$\bar{i}_{max} = 28.9\ \mu a$

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